G. Sahut (LEGI), G. Balarac, V. Moureau, G. Lartigue, P. Bénard, A. Grenouilloux

Implicitation of Navier-Stokes equations

$$\frac{u^{n+1}-u^n}{\Delta t}+\nabla\cdot\left(u^{n+1}u^{n+1}\right)=-\frac{\nabla p^{n+\frac{1}{2}}}{\rho}+\nabla\cdot\tau^{n+1}$$

Steady state

loop over n

$$\frac{u^*-u^n}{\Delta\tau}+\nabla\cdot(u^n u^*)=-\frac{\nabla p^{n-\frac{1}{2}}}{\rho}+\nabla\cdot\tau^*$$

$$\frac{\nabla\cdot u^*}{\Delta\tau}=\frac{1}{\rho}\nabla^2\left(\delta p^{n+\frac{1}{2}}\right),\quad \delta p^{n+\frac{1}{2}}=p^{n+\frac{1}{2}}-p^{n-\frac{1}{2}}$$

$$\frac{u^{n+1}-u^*}{\Delta\tau}=-\frac{\nabla\left(\delta p^{n+\frac{1}{2}}\right)}{\rho}$$

until $\left\|u^{n+1}-u^n\right\|<\epsilon$

Unsteady state

$$u^{k=0}=u^n$$
$$p^{k=0}=p^n$$

loop over k

$$\Delta\tau\rightarrow\Delta t \qquad \delta p^{n+\frac{1}{2}}=\delta p^{k+\frac{1}{2}}$$
$$u^n\rightarrow u^k$$
$$u^*\rightarrow u^{k,*}$$
$$u^{n+1}\rightarrow u^{k+1}$$

until $\left\|u^{k+1}-u^k\right\|<\epsilon$

$$u^{n+1}=u^{k+1}$$

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Steady state

loop over n

$$\frac{u^{*}-u^{n}}{\Delta\tau}+\nabla\cdot\left(u^{n}u^{*}\right)=-\frac{\nabla p^{n-\frac{1}{2}}}{\rho}+\nabla\cdot\tau^{*}$$

$$\frac{\nabla\cdot u^{*}}{\Delta\tau}=\frac{1}{\rho}\nabla^{2}\left(\delta p^{n+\frac{1}{2}}\right),\quad \delta p^{n+\frac{1}{2}}=p^{n+\frac{1}{2}}-p^{n-\frac{1}{2}}$$

$$\frac{u^{n+1}-u^{*}}{\Delta\tau}=-\frac{\nabla\left(\delta p^{n+\frac{1}{2}}\right)}{\rho}$$

until $\left\|u^{n+1}-u^{n}\right\|<\epsilon$

New implicit_incompressible solver

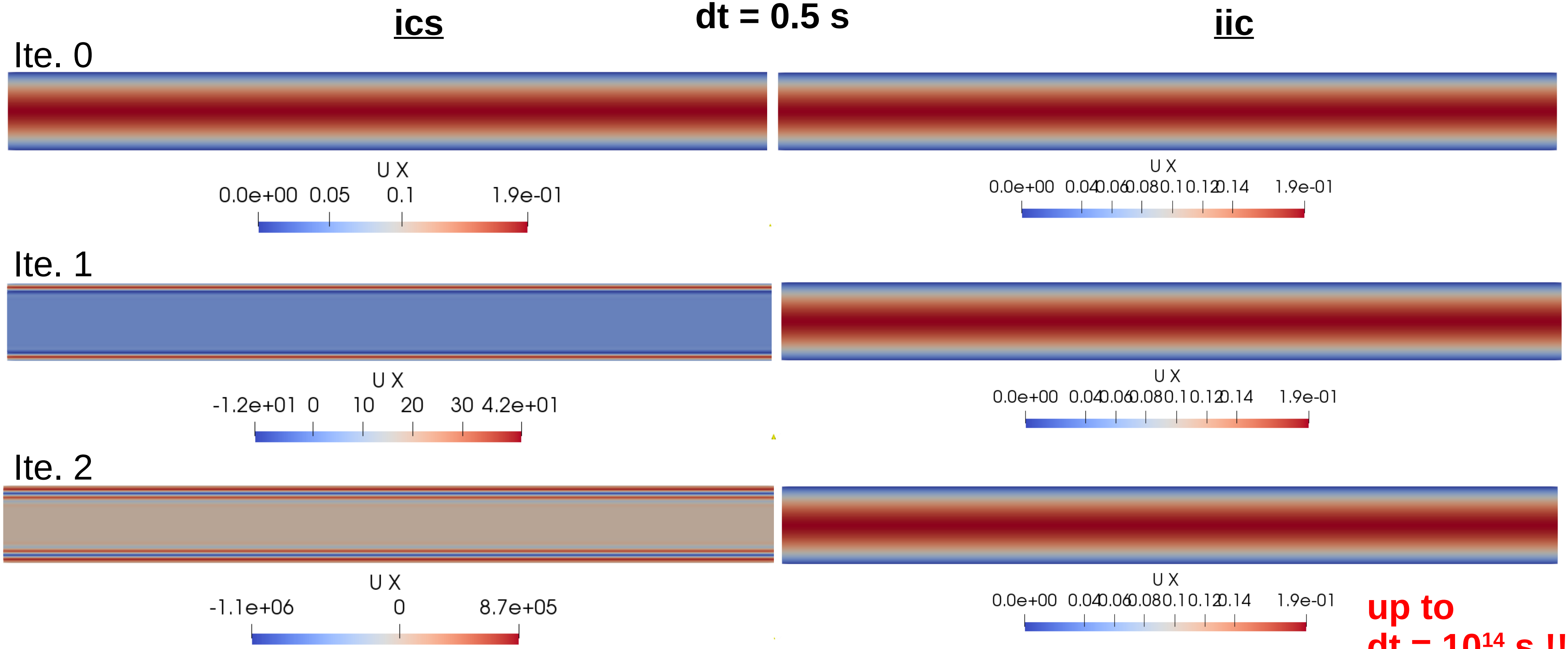
➡ based on ics and hts solvers

➡ 2 linear systems per iteration

- $\nabla\cdot\tau^{*}=\nu\nabla^{2}u^{*}$ similar to $\lambda\nabla^{2}T^{n+1}$ in hts
- $\nabla\cdot\left(u^{n}u^{*}\right)$ ⟶ asym_op_val(ip) = 0.5_WP*udotAp

Validation : 2D Poiseuille test case with PERIODIC boundary conditions

dt = 0.5 s



up to dt = 10^{14} s !!

Implicitation of scalars extended from hts (A. Grenouilloux)

$$\frac{\partial \phi}{\partial t} + \underbrace{\nabla \cdot (\phi u)}_{\text{linear term}} = \underbrace{D\nabla^2\phi}_{\text{linear term, similar to } \lambda\nabla^2 T^{n+1} \text{ in hts}}$$

Perspectives for the iic solver

- integrate the implicitation of scalars
- implement implicit RANS models
- handle more complex boundary conditions